California High-Speed Train Project

TECHNICAL MEMORANDUM

Alignment Design Standards for High-Speed Train Operation TM 2.1.2

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System Level Technical and Integration Reviews

The purpose of the review is to ensure:

- Technical consistency and appropriateness
- Check for integration issues and conflicts

System level reviews are required for all technical memoranda. Technical Leads for each subsystem are responsible for completing the reviews in a timely manner and identifying appropriate senior staff to perform the review. Exemption to the System Level technical and integration review by any Subsystem must be approved by the Engineering Manager.

System Level Technical Reviews by Subsystem:

Systems:	*Signed document on file*_________ Eric Scotson	_19 Feb 09 _ Date
Infrastructure:	*Signed document on file*_________ John Chirco	_17 Mar 09 _ Date
Operations:	*Signed document on file*_________ Paul Mosier	_05 Mar 09 _ Date
Maintenance:	*Signed document on file*_________ Paul Mosier	_05 Mar 09 _ Date
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ABSTRACT

This technical memorandum presents alignment design guidance for the segments of the proposed high-speed rail line in exclusive use operation. On these segments, speeds will be above 125 mph (200 km/h) up to a maximum operating speed of 220 mph (350 km/h) and will consider that faster operation up to not less than 250 mph (400 km/h) in the future will not be unnecessarily precluded. The technical memorandum defines the geometric design requirements for basic design in order to achieve a safe and reliable operating railway that meets regulatory and CHSTP functional; programmatic, operational and performance requirements.

The general basis of alignment design will be to follow best practices of the Japanese and European lines and also the guidance of UIC (International Union of Railways) for railway lines, while also taking into account common American practices and the guidance of the Manual for Railway Engineering of the American Railway Engineering and Maintenance of Way Association (AREMA Manual).

Guidance for the design of alignments in shared used corridors where train speeds will be equal to or less than 125 mph (200 km/h) are provided in the Technical Memorandum 1.1.6 – Alignment Standards for Shared Use Corridors (Specific to Los Angeles to Anaheim). Similar alignment guidance for the Caltrain corridor is under development.

Guidance for turnout geometry will be provided in a separate document.

Guidance for parking, maintenance and siding tracks will be provided in a separate document. However, some information on the location of passing tracks which are parallel to the main tracks is covered in this document.

6.0 DESIGN MANUAL CRITERIA

6.1 ALIGNMENT DESIGN

The following information applies to dedicated high-speed lines where the high-speed trainsets are the sole rolling stock in use. The Corridor shall be exclusively used by equipment designed and constructed for high-speed operation above 125 mph (200 km/h) and shall be designed to a set of criteria specific to that purpose.

The basic principle in alignment design is to provide the smoothest line as practical by minimizing frequency and severity of changes in direction and profile. Where changes in direction and profile occur, they should be as gentle as practical. Over four changes in direction per mile shall constitute an Exceptional condition.

6.1.1 Minimum Lengths of Alignment Segments

Attenuation time, based on the most conservative requirements, shall be:

- For V < 300 km/h (Under 186 mph)
  - Desirable attenuation time: not less than 2.4 seconds
  - Minimum attenuation time: not less than 1.8 seconds
  - Exceptional attenuation time: not less than 1.5 seconds
  - An attenuation time of 1.0 seconds on the diverging route in curves adjacent to or between turnouts
- For 300 km/h ≤ V (Over 186 mph)
  - Desirable attenuation time: not less than 3.1 seconds
  - Minimum attenuation time: not less than 2.4 seconds
  - Exceptional attenuation time: not less than 1.8 seconds

Minimum segment length is calculated by the formula: $L_{feet} = V_{mph} \times 44/30 \times t_{sec}$ and $L_m = V_{km/h} / 3.6 \times t_{sec}$. Minimum segment lengths are presented in Tables 6.1.1 and 6.1.2.

Table 6.1.1: Minimum Segment Lengths at Various Speeds

Design Speed		Minimum Segment Lengths for times of							
		3.1 seconds		2.4 seconds		1.8 seconds		1.5 seconds	
miles per hour	km/h	feet	meters	feet	meters	feet	meters	feet	meters
250	400	1137	346	880	268	660	201	550	168
220	355	1000	305	774	236	581	177	484	148
200	320	909	277	704	215	528	161	440	134
186	300	846	258	655	200	491	150	409	125
175	280	796	243	616	188	462	141	385	117
150	240	682	208	528	161	396	121	330	101

Table 6.1.2: Minimum Segment Lengths at Various Speeds

Design Speed		Minimum Segment Lengths for times of							
		2.4 seconds		1.8 seconds		1.5 seconds		1.0 seconds	
miles per hour	km/h	feet	meters	feet	meters	feet	meters	feet	meters
186	300	655	200	491	150	409	125	273	83
175	280	616	188	462	141	385	117	257	78
150	240	528	161	396	121	330	101	220	67
125	200	440	134	330	101	275	84	183	56

Where alignment segments overlap, each change shall be treated as a separate alignment element for the purpose of calculating minimum segment lengths. For example, where there is a vertical curve within a horizontal curve, the parts of the horizontal curve outside of the vertical curve will be treated as separate segments when calculating segment lengths.

The segment length requirement will govern only where other design considerations for the various alignment elements do not require longer segment lengths.

6.1.2 Minimum Radii

As in segment lengths, curve radii standards will be provided for “Desirable”, “Minimum”, and “Exceptional” values. However, even the “Desirable” figures can be improved upon. A “Desirable” radius is one that is larger than this radius if at all practical, for larger radii are better. The radii provided in the table are based on superelevation calculation described in paragraph 6.1.4: The EU Technical Specification for Interoperability has a break in allowable unbalanced superelevation at 300 km/h (186 mph), thereby resulting in two values being given for the speed of 186 mph.

Table 6.1.3: Minimum Curve Radii

Design Speed		**Minimum radius, Based on Superelevation Limits**					
		Desirable		**Minimum**		**Exceptional**	
miles per hour	km/h	feet	meters (rounded)	feet	meters (rounded)	feet	meters (rounded)
250	400	45,000	13,700	28,000	8,500	25,000	7,600
220	355	35,000	10,700	22,000	6,700	19,500	6,000
200	320	30,000	9,200	18,000	5,500	16,000	4,900
186	300	25,000	7,600	16,600	4,700	14,000	4,250
<186	<300	25,000	7,600	16,600	4,700	12,600	3,850
175	280	22,000	6,700	14,000	4,200	11,200	3,400
150	240	16,000	4,900	10,000	3,100	8,200	2,500
125	200	10,500	3,200	7,000	2,100	5,700	1,750

For those used to working in degrees of curve (railroad definition), the following degrees of curvature may be used as substitute values for the radii given in Table 6.1.3. Please note that the values in Table 6.1.4 are not exact conversions of the radii given in Table 6.1.3, but are convenient values for degree of curve limits for the same speed standards.

Table 6.1.4: Minimum Degree of Curve

Design Speed		Minimum Curve in Degrees, degree, minutes, seconds		
miles per hour	km/h	Desirable	Minimum	Exceptional
250	400	0d 07m 30s	0d 12m 15s	0d 13m 30s
220	355	0d 09m 45s	0d 15m 30s	0d 17m 30s
200	320	0d 11m 15s	0d 19m 00s	0d 21m 15s
186	300	0d 13m 45s	0d 21m 30s	0d 24m 30s
<186	<300	0d 13m 45s	0d 21m 30s	0d 27m 15s
175	280	0d 15m 30s	0d 24m 30s	0d 30m 30s
150	240	0d 21m 15s	0d 34m 15s	0d 41m 45s
125	200	0d 32m 30s	0d 49m 00s	1d 00m 00s

At high speeds the distance between curves is determined by the minimum segment length. Guidance for low speed tracks is provided in California High-Speed Train Project Technical Memorandum 1.1.6 - Alignment Standards for Shared Use Corridors (Specific to Los Angeles to Anaheim).

Curves with Small Central Angles: For small central angles the radius shall be sufficiently large to provide the time-based minimum arc and spiral segment lengths. There is no maximum radius requirement or desirable maximum for radius. In general, larger radii are preferable to smaller radii as the superelevation and unbalance values become smaller. A radius should be selected that results in the length of the simple curve portion being about equal to or longer than the length of spiral. Since each portion is an alignment segment, if each segment is equal in length, the entire curve with spirals should have a minimum length not less than three times the Minimum Segment Length given in Table 6.1.1 or 6.1.2, as appropriate for the design speed. Double spirals or curves with long spirals and short arc lengths shall not be used.

6.1.3 Superelevation

Balancing superelevation shall be calculated by one of the following formulae, depending upon how the curve is defined:

$$SE = 0.0007\ V^2\ D$$ (curvature in degrees, speed in mph and SE in inches)

Which when expressed with radius instead of degrees gives:

$$SE = 4.0\ V^2 / R$$ (radius in feet, speed in mph and SE in inches) or

Or in metric system units:

$$SE = 11.8\ V^2 / R$$ (radius in meters, speed in km/h and SE in millimeters)

Curves shall not be superelevated to balance the design speed, calculated average speed, or maximum operating speed. A certain amount of unbalance, usually 1.0 inches at normal operating speed, is desirable for ride comfort and smooth running of the vehicles through the curve.

The design value of superelevation to be applied to the curve will be influenced by:

- Maximum Speed Limit
- Calculated normal and maximum speeds of high-speed trains
- Where applicable, calculated normal and maximum speeds of high-speed trains slowing for or accelerating from a station stop
- Calculated normal and maximum speeds of other passenger trains (where applicable)
- Minimum unbalance of 1.0 inch (25 mm) in relation to the normal train speed.

Design superelevation shall not exceed the allowable maximum superelevation. Design superelevation shall be calculated for each track. It is neither necessary nor in many locations desirable that both main tracks of the line have the same superelevation on a given curve. Where the normal operating speed of the trains differ from the Design Speed, the unbalanced superelevation may be increased up to the Exceptional level based on the Design Speed if it is necessary to provide a comfortable unbalance for trains moving at their normal operating speeds.

The first of the following tables provide the allowable upper limits for Superelevation plus Unbalanced Superelevation. Radii developed from these limits determine the Desirable, Minimum, and Exceptional radius that is permissible for any given speed.

Table 6.1.5: Maximum Values, Superelevation plus Unbalanced Superelevation

Design Speed		Combined Superelevation and Unbalance					
		Desirable		Maximum		Exceptional	
miles per hour	km/h	inches	mm	inches	mm	inches	mm
<186	<300	6	150	9	230	11	280
≥186	≥300	6	150	9	230	10	250

Table 6.1.6: Maximum Values of Applied Superelevation

Design Speed		Applied Superelevation					
		Desirable		Maximum		Exceptional	
miles per hour	km/h	inches	mm	inches	mm	inches	mm
<186	<300	4	101	6	150	7	180
≥186	≥300	4	100	6	150	7	180

Table 6.1.6: Maximum Values of Unbalanced Superelevation

Design Speed		Unbalanced Superelevation					
		Desirable		Maximum		Exceptional	
miles per hour	km/h	inches	mm	inches	mm	inches	mm
<186	<300	2	50	3	75	4	100
≥186	≥300	2	50	3	75	3	75

6.1.5 Spirals

6.1.5.1 Selection of Speed to be Used in Design

The speed to be used at the design stage of the alignment is the system design speed, not the operating speed planned to be used at the time of start of operations. During alignment design, the purpose of determining superelevation is to find the appropriate spiral length for a particular curve in the alignment design. Thus, the highest anticipated speed, superelevation and unbalanced superelevation shall be used at this stage of the design.

However, in construction of the track, the superelevation shall be based on the calculated speed on the curve on the time of initial operation. Therefore, the initial superelevation applied to the track may be less than that developed in the calculations for appropriate spiral length.

6.1.5.2 Selection of Spiral Type

In ballasted tracks a certain amount of "transition into the transition" naturally occurs with the normal maintenance processes. In concrete slab supported tracks this "transition into the transition" cannot occur. Therefore, it is desirable to change from constant rate spirals to variable rate spirals at a lower speed where using concrete base trackforms. For concrete base trackforms, the point of change will be set in accordance with Japanese Shinkansen practice. For ballasted track, a higher speed may be used.

Half-Sine Spirals (variable rate transitions) shall be used on all tracks designed for:

- Ballasted tracks: Curves having design maximum speeds of 80 mph or more
- Non-ballasted tracks: Curves having design maximum speeds of 60 mph or more
- Curves associated with turnouts having design maximum speeds of 110 mph or more

Clothoid Spirals (constant rate transitions) shall be used on all lower speed tracks. Clothoid spirals may also be used on very large radius curves that require small amounts or no superelevation and have very small unbalanced superelevations, as described below under special situations.

Nature of the Internal Transitions in the Two Spiral Types:

Half Sine Spiral (Angles in these formulae are in radians.):

Local Radius through the Spiral:

$$R_{loc} = 2\ R_{curve} / (1 - \cos(\pi\ L_{loc} / L_{tot}))$$

Local Superelevation through the Spiral:

$$SE_{loc} = 0.5\ SE_{curve}\ (1 - \cos(\pi\ L_{loc} / L_{tot}))$$

Clothoid Spiral:

Local Radius through the Spiral:

$$R_{loc} = R_{curve} / (L_{tot} / L_{loc})$$

Local Superelevation through the Spiral:

$$SE_{loc} = SE_{curve}\ (L_{loc} / L_{tot})$$

6.1.5.3 Determining Spiral Length

Spiral Lengths: The length of the spiral shall be the longest length determined by calculating the various length requirements, which are:

- Length needed to achieve Attenuation Time
- Length determined by allowed rate of change in superelevation
- Length determined by allowed rate of change in unbalanced superelevation
- Length determined by limitation on twisting over vehicle and truck spacing length

Table 6.1.7: Minimum Length of Spiral

Half-Sine (Variable Change) Spirals *			
Spiral Design Factor	Desirable	Minimum	Exceptional
Superelevation	1.63 Ea V	1.30 Ea V	1.09 Ea V
Unbalance	2.10 Eu V	1.57 Eu V	1.26 Eu V
Twist **	140 Ea	118 Ea	98 Ea
Minimum Segment	2.64 V	2.20 V	1.47 V
Clothoid (Linear Change) Spirals			
Spiral Design Factor	Desirable (0.03 g)	Minimum (0.04 g)	Exceptional (0.05 g)
Superelevation	1.47 Ea V	1.17 Ea V	0.98 Ea V
Unbalance	1.63 Eu V	1.22 Eu V	0.98 Eu V
Twist	90 Ea	75 Ea	62 Ea
Minimum Segment	2.64 V	2.20 V	1.47 V

The length is given in feet with:

- Ea = Actual elevation in inches
- Eu = Unbalanced elevation in inches
- V = maximum speed of the train in mph

* Longer lengths of half-sine spirals are due to the variability in the ramp rate.

** Provides maximum twist rates identical to clothoids. As a practical matter, this limitation never governs due to use of this type spiral only on high-speed tracks.

After calculation and selection of length, based on the governing requirement, the spiral length should then be rounded to a convenient value for further calculation and use in the alignment. Rounding may be either up or down for “Desirable” values so long as the downward rounding does not reduce any of the required desirable lengths by more than 5.0%. Rounding may be either up or down for “Minimum” values so long as the downward rounding does not reduce any of the required minimum lengths by more than 1.0%. Rounding shall only be in the upward direction for “Exceptional” values.

6.1.5.4 Special Situations

Spirals on Large Radius Curves: Clothoid spirals may be used instead of half-sine spirals regardless of track type or design speed if the following conditions are met: The required superelevation and unbalanced superelevation are both under 1.0 inches at the maximum design speed; and the “Minimum Segment” length for the spiral is more than twice the length required by any other factor. Spirals may be omitted if the following conditions are met: The required superelevation is zero (balancing superelevation for the maximum speed less than 0.75 inches); and the calculated offset of the curve due to application of the spiral is less than 0.05 feet in ballasted track or less than 0.02 feet in non-ballasted track. *(These values are subject to revision.)*

Reverse Curves: If there is insufficient distance between curves to provide the minimum required length tangent segment, the spirals shall be extended to provide a reversing curve. If beneficial to design and construction, a straight distance between curves that would be run in less than 0.2 seconds at the normal operating speed may be left between spiral ends.

6.1.5.5 Sample Calculation

Determine the spiral length for a 50,000 foot radius curve in the main line portion where the design speed will be 250 mph and the initial operating speed limit will be 220 mph, and located where it will not be affected by lower speeds of some or all trains due to acceleration, braking, or grades.

At 250 mph, balancing superelevation will be 5.00 inches. The 90% speed is 225 mph, which balances at 4.050 inches.

At 220 mph, balancing superelevation will be 3.872 inches. The 90% speed is 198 mph, which balances at 3.126 inches.

Spiral Length will be based on 3.00 inches superelevation and 2.00 inches unbalance for the 250 mph situation.

Since the initial maximum operating speed will be 220 mph, the actual installed superelevation upon opening will be 2.125 inches, giving an initial unbalance of 1.001 inches. Since it is impractical to change spiral length, the spiral length will not be based on this 220 mph design, but shall be determined based on the 250 mph design speed.

The Spiral type will be Half-Sine

Desirable Design Factors, based on 250 mph:

Superelevation: 1.63 Ea V = 1.63 x 3.00 x 250 = 1222.5 feet

Unbalance: 2.10 Eu V = 2.10 x 2.00 x 250 = 1050 feet

Obviously, neither twist nor minimum segment length need be considered for this spiral.

Select a Length of **1,200 feet** for this spiral. (less than 1.9% under the calculated length.)

6.1.6 Grades and Vertical Curves

The railroad alignment shall be designed to have the smoothest practical profile while optimizing earthwork, structures, tunnels and drainage. Grades shall be as low as practical. Where grades do occur, they should be of the same slope from bottom to top where practical. Use of multiple short grades and multiple changes in grade within any particular change of elevation ("sawtooth profiles") are to be avoided to the greatest extent practical. In addition to increasing operational costs and difficulty by requiring frequent changes in power, a line with multiple changes in grade is aesthetically unappealing. As a check on the reasonableness of the profile developed, it shall be drawn up at a highly condensed horizontal scale so that the vertical changes are exaggerated. Otherwise, the alignment can appear deceptively smooth.

Low points and very flat grades should not be used in cuts or tunnels (including Cut and Cover) due to drainage considerations.

6.1.6.1 Limitations on Grades

Maximum Grade Limits:

- Desirable grades: as low as reasonably practical, with a limit of 1.25%
- Maximum grades: above 1.25% and shall be as low as practical up to 2.50%
- Exceptional grades: above 2.50% and shall be as low as practical up to 3.50%

Minimum Grades: Without a separate drainage system, grades in cuts or tunnels (included cut-and-cover) shall not be less than 0.25%.

Limitation on Length of Steep Grades: Where terrain permits, long grades steeper than the following shall not be used due to limits of breaking capability of some of the proposed train sets:

- The average grade for any 6 km (3.7 mi) long section of the line shall be under 3.5%
- The average grade for any 10 km (6.2 mi) long section of the line shall be under 2.5%

Limitations of Speed on Grades: In European practice, speed on downgrades is constrained by train set braking limitations. The restriction is based on the average grade over any continuous length of 5,200m (17,100ft) along the line. The following speed limits for different grades are as determined in accordance with French standards:

- Grade between 3.0% and 3.5%: Vmax = 230 km/h (143 mph)
- Grade between 2.2% and 3.0%: Vmax = 270 km/h (168 mph)
- Grade between 1.6% and 2.2%: Vmax = 300 km/h (186 mph)
- Grade between 0.0% and 1.6%: Vmax = 350 km/h (217 mph)

Limitations on Grade at power supply phase breaks: In European practice, due to this catenary and signaling constraints it is desirable to limit grades to 0.60% for 600 m (2,000 feet) on each side of a phase break. The need for this constraint and its locations shall be confirmed in consultation with the development of design standards for the electrification system.

6.1.6.2 Vertical Curves

Normal US Practice

In US railroad alignment, vertical curves are parabolic curves normally defined by the rate of change in grade per 100 feet of length. This is the inverse of the usage in highway work, which also uses parabolic curves, but defined by the length required for a one percent change in grade.

Normal Japanese and European Practice

Normal practice in Japan and Taiwan is also to use Parabolic Vertical Curves. Even though the vertical curves on the Taiwan High-Speed Railway were defined by radius, they were designed and constructed as parabolas.

Common European practice in both highway and railroad work is to use circular radius vertical curves. Consequently, European high-speed railways are normally designed and built with circular vertical curves.

The radius of the curve at the crest or sag is determined in accordance with the vertical acceleration permitted for passenger comfort and the maximum speed of the line. The formula in metric units is: $R_{min} \geq (V/3.6)^2 / a_v$, where R is in meters, V in km/h, Vertical acceleration (av) in m/s^2 and the 3.6 factor is necessary for the km/h to m/s conversion. The formula in US Customary units would be: $R_{min} \geq (V*44/30)^2 / a_v$, where R is in feet, V in mph, Vertical acceleration (av) in $feet/sec^2$ and the 44/30 is necessary for the mph to ft/sec conversion.

European high-speed lines do not use curves with radius over 40,000m (131,234 feet radius, or a rate of change 0.0762% per 100 feet) due to maintainability concerns.

In SNCF practice, it is unnecessary to provide a vertical curve between two gradients when the difference in gradient is less than or equal to 0.1% or 1mm/m, even for design speeds of 350 km/h (217 mph).

Comparison between American and European Vertical Curve Design Practices:

The difference between arc vertical curves and parabolic vertical curves as laid out is very slight, usually under $^1/_8$ inch (3 mm).

Differences in maximum vertical curve radius requirements, or in US Customary railroad terminology, minimum rate of change requirements:

- It is reported that European measuring systems and maintenance practices are incapable of measuring vertical curves with radii of over 40,000 meters (0.0762% grade change per 100 feet), therefore vertical curves longer than this rate of change are prohibited.
- Rates of change down to 0.05 percent per 100 feet (Radius = 60,100 m) have been used in the US for many years without any maintainability issues ever being raised.
- The AREMA Manual states simply, "Vertical curves should be designed as long as physically and economically possible."

- The Taiwan High-Speed Railway has multiple vertical curves with radii of 100,000 meters to 300,000 meters (rate of change of 0.03 %/100ft to 0.01 %/100ft).
- It is anticipated that the high-speed tracks will be of some form of concrete slab type construction. Therefore, even if the limitations of European maintenance machinery were of concern, they will not be an issue.
- Conclusion: No upper limit needs to be set on vertical curve length/radius.

The relationship between vertical curves defined by radius and vertical curves defined by rate of change is as follows:

- Rate of change, %/100 feet = 3048% / Radius in meters
- Radius in meters = 3048% / (%/100 feet)
- Example: 40,000 meters: 3048% / 40,000 = 0.0762 percent per 100 feet
- Rounding: If a 40,000 m radius is desired, then a rate of change of 0.075 percent per 100 feet or less would be a reasonable value to use to determine the length of vertical curve.

Vertical Curve Type Shall be Parabolic

Parabolic vertical curves shall be used in order to be in line with common US practice.

The speed-based formula for vertical curve length in the AREMA Manual provides a length of vertical curve that is equivalent to one based on change in grade and radius derived by using the vertical acceleration-based formula

In line with common US railroad practice, the length of vertical curves shall be rounded to nearest 100 foot increment where practical.

Vertical Curve Acceleration Rates:

The acceleration values to be used for vertical curves shall be:

- Desirable: 0.60 ft/sec/sec (1.86 percent of gravity) – AREMA recommended practice for passenger railroads.
- Minimum: 0.90 ft/sec/sec (2.80 percent of gravity)
- Exceptional: 1.40 ft/sec/sec (4.35 percent of gravity)

Vertical curve lengths on lines carrying high-speed trains only shall be:

- Desirable VC Length: The longer of $LVC_{feet} = 4.55\ V$ (for 3.1 seconds) or $LVC_{feet} = 2.15\ V^2\ (\Delta\% / 100) / 0.60\ ft/sec^2$, but not less than 400 $\Delta\%$
- Minimum VC Length: The longer of $LVC_{feet} = 3.52\ V$ (for 2.4 seconds) or $LVC_{feet} = 2.15\ V^2\ (\Delta\% / 100) / 0.80\ ft/sec^2$, but not less than 200 $\Delta\%$
- Exceptional VC Length: The longer of $LVC_{feet} = 2.64\ V$ (for 1.8 seconds) or $LVC_{feet} = 2.15\ V^2\ (\Delta\% / 100) / 1.20\ ft/sec^2$, but not less than 100 $\Delta\%$
- The speed used in the preceding formulae shall be no less than 250 mph, except where other alignment factors such as speed limiting curves exist. In those locations, a lower speed equal to or higher than the maximum anticipated achievable train speed may be used to calculate the required vertical curve lengths. At 250 mph, these formulae give:
  - Desirable VC Length: $LVC_{feet} = 2250\ \Delta\%$
  - Minimum VC Length: $LVC_{feet} = 1500\ \Delta\%$
  - Exceptional VC Length: $LVC_{feet} = 970\ \Delta\%$

The 2.15 factor is a constant necessary to unit conversions within the US Customary measuring system.

Tables 6.1.8-1 and 6.1.8-2 illustrate the relationship between the various methods of calculating vertical curves. All values are rounded.

Table 6.1.8-1: Desirable Vertical Curves – Rates of Change and Equivalent Radii (0.60 ft/s^2 = 1.86% g)

Speed mph	Speed km/h	% change per 100 feet	feet per % of change	Radius feet	Radius meters
300	480	0.030%	3250	325,000	100,000
250	400	0.045%	2250	225,000	70,000
220	355	0.060%	1750	175,000	53,000
200	320	0.070%	1450	145,000	44,000
175	280	0.090%	1100	110,000	33,000
150	240	0.120%	810	81,000	25,000
125	200	0.175%	560	56,000	17,000

Table 6.1.8-2: Minimum Vertical Curves – Rates of Change and Equivalent Radii (0.90 ft/s^2 = 2.80% g)

Speed mph	Speed km/h	% change per 100 feet	feet per % of change	Radius feet	Radius meters
300	480	0.045%	2150	215,000	66,000
250	400	0.065%	1500	150,000	46,000
220	355	0.085%	1160	116,000	36,000
200	320	0.100%	960	96,000	30,000
175	280	0.130%	740	74,000	22,500
150	240	0.180%	540	54,000	16,500
125	200	0.260%	375	37,500	11,500

Table 6.1.8-3: Exceptional Vertical Curves – Rates of Change and Equivalent Radii (1.4 ft/s^2 = 4.35% g

Speed mph	Speed km/h	% change per 100 feet	feet per % of change	Radius feet	Radius meters
300	480	0.070%	1400	140,000	43,000
250	400	0.100%	970	97,000	30,000
220	355	0.130%	750	75,000	23,000
200	320	0.150%	620	62,000	19,000
175	280	0.200%	480	48,000	15,000
150	240	0.250%	350	35,000	11,000
125	200	0.400%	250	25,000	7,500

The lengths developed in the preceding tables and formulae are the shortest allowed lengths in each scenario. Vertical curve lengths shall always be rounded up. Rate of Change, radius and other parameters of the vertical curve shall then be derived from this length.

Where the difference between gradients is small, the minimum segment length requirements described in Section 6.1.1 shall determine the minimum length of vertical curve. Rate of Change, radius and other parameters of the vertical curve shall then be derived from this length.

Where CHSTP lines closely parallel lines for other passenger or freight trains such that a common profile is desirable, the longest vertical curve length determined by the separate calculation for each type of traffic shall determine the vertical curve length to be used for all tracks.

6.1.7 Horizontal Curves in Vertical Curves

Unbalanced Superelevation Limits: Horizontal and vertical curves can overlap. Crest vertical curves result in a downward acceleration of the vehicle, thereby reducing the gravitational effect. This reduction is small but not insignificant for the vertical curve rates of change permitted in this document. A reduction of 0.25 inches for limiting and 0.50 inches for exceptional unbalanced superelevation is sufficient to allow for this effect.

Vertical Curves in Spirals: Due to potential maintenance difficulties, it is desirable to avoid use of vertical curves in spirals. The desirable distance between end of spiral and beginning of vertical curve or end of vertical curve and beginning of spiral is 160 feet (50 m) with a minimum limit of 100 feet (30m). Overlap between vertical curves and spirals may be permitted as an Exceptional condition, but only where it can be shown that practical alternatives have been exhausted.